Technical Notes

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Effect of Ambient Pressure on Nozzle Centerline Flow Properties

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An assumption that is sometimes made with regard to the expansion of a nozzle flow into a vacuum is that the static pressure p_∞ across the exit plane is constant at the centerline value. Furthermore, it has been demonstrated that, provided the background pressure p_{bg} is maintained at a level less than or equal to the nozzle centerline static pressure, then no recompression shocks will occur in the nozzle exit plane. This has been defined as the underexpanded mode of nozzle operation. Operation in the overexpanded mode, where the background pressure is greater than the nozzle centerline static pressure, results in compression shocks in the nozzle exit plane. Even though these shocks can exert an upstream influence in the nozzle boundary layer, the central core of the flow is unaffected and remains isentropic. It is the purpose of this Note to show that, for significant reductions in background pressure below the exit plane centerline value, the nozzle centerline flow properties are affected.

An experimental test program sponsored by the Air Force Rocket Propulsion Laboratory is currently being performed at the Arnold Egineering Development Center to define the flow characteristics of the plume that result when a nozzle flow expands into a hard vacuum. In one phase of this test program, measurements have been made of the radial pitot pressure profiles in the nozzle exit plane as a function of chamber background pressure. For overexpanded flow, i.e., $p_{bg} > (p_\infty)_{\mathrm{CL}}$, the presence of the recompression shock was clearly indicated by large off-axis spikes in the pitot pressure profiles. For underexpanded flow conditions, i.e., $p_{bg} < (p_\infty)_{\mathrm{CL}}$, there is a small but consistent increase in the diameter of the isentropic core flow (associated with a corresponding decrease in nozzle boundary-layer thickness) as the background pressure decreases. For constant stagnation conditions, i.e., $p_0 = 450$ Torr and $T_0 = 475$ K, this changing core diameter was associated with a decrease in the value of the pitot pressure, which implies an increase in centerline Mach number. Mach number has been derived from the ratio of pitot pressure to stagnation pressure with the assumption that the flow is isentropic. The variation of centerline Mach number with chamber background pressure is presented in Fig. 1.

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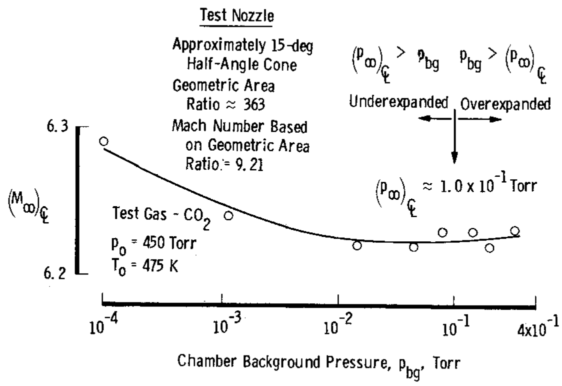

Fig. 1 Exit plane centerline Mach number as a function of chamber background pressure.

An earlier study by Rothe[1] indicated that the centerline flowfield properties were unaffected by changes in chamber background pressure. However, Rothe's experiments were limited to chamber background pressures that were one-fifth of the centerline static pressure. For this range of chamber pressures, the present investigation also does not show a measurable change in centerline Mach number. This suggests that had Rothe been able to reduce chamber pressure by a factor of 100 rather than 5, he, too, may have observed an effect upon centerline flow properties.

If, as the present experimental measurements show, the nozzle boundary-layer thickness (and hence nozzle centerline flowfield properties) is affected by changes in p_{bg} when $p_{bg} < (p_\infty)_{\mathrm{CL}}$, then the following consequences may need to be addressed. Measurements of nozzle thrust characteristics in ground-based test facilities may not be wholly representative of the flight values if these tests are not carried out at the actual flight values of ambient pressure. If in the course of a ground-based evaluation of nozzle performance the ambient pressure changes, it may not be completely accurate to assume that the nozzle flow characteristics remain constant. A change such as this could have an effect upon the boundary-layer expansion into the backflow region in pulsed motor tests of the type described by Williams et al.[2] In studies of flow in the nozzle lip region, Bird[3] has suggested that the sonic line in the nozzle boundary layer intersects the nozzle lip and thus effectively insulates the nozzle boundary layer from external influences. Thus, codes that have been developed or are being developed to predict the expansion characteristics of the nozzle boundary layer into the backflow region of the nozzle should be able to account for ambient pressure nozzle boundary-layer interaction effects.

Acknowledgment

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References

[1]Rothe, D. E., "Electron-Beam Studies of Low Reynolds Number Flows Inside a DeLaval Nozzle," AIAA Paper 70-810, June 1970.

[2]Williams, W. D. et al., "Experimental Study of the Plume Characteristics of an Aged Monopropellant Hydrazine Thruster," AEDC-TR-79-2 (AD-A068326), April 1979.

[3]Bird, G. A., "The Nozzle Lip Problem," *Proceedings of the Ninth International Symposium on Rarefied Gas Dynamics*, DFVLR Press, Porz-Wahn, FRG, 1974, Vol. I, pp. A22-1–A22-8..

Calculation of the Volume of a General Hexahedron for Flow Predictions

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Introduction

FOR application of finite volume methods to the solution of conservative partial differential equations the volume of elementary cells must be calculated. The elementary cells are usually general hexahedra with the eight corners of each cell prescribed. The 12 sides of each cell are straight lines joining these corners and therefore the faces are bounded by quadrilaterals, which need not be planar. In previous work (see e.g., Ref. 1) the faces have been defined by two plane triangles obtained after inserting a diagonal in each quadrilateral and then the volume of the cell has been determined. In general, 64 different such cells can be formed, each with a different volume, because there is a choice of two diagonals per face.

A different construction of the cell is to take a face bounded by a quadrilateral to be a doubly-ruled surface. There is one particular doubly-ruled surface that is rather special in that cells with these surfaces can be fitted together to fill the whole space without any consideration of matching of diagonals of the face quadrilaterals. Only one such cell can be constructed once the bounding quadrilaterals have been set up; hence, only one value of volume is obtained. There is no discontinuity of slope or curvature anywhere on the faces of this cell. In this Note an expression for the volume of such a general hexahedral cell is obtained.

Calculation of Pyramid Volume with a Base Formed of a Doubly-Ruled Surface Bounded by a Quadrilateral

The general hexahedron is divided into six pyramids with a common vertex and with one of the faces as the base of each. Consider one of these pyramids with vertex R and base formed of a doubly-ruled surface bounded by the quadrilateral ABCD as shown in Fig. 1. The side-faces RAB, RBC, RCD, and RDA are plane.

Take the vector displacement **RP** of point P from point R when point P lies on the base ABCD of the pyramid to be given by

$$\mathbf{RP} = \mathbf{RA} + \xi\mathbf{AB} + \eta\mathbf{AD} + \xi\eta(\mathbf{AC} - \mathbf{AB} - \mathbf{AD}) \tag{1}$$

where ξ, η are surface curvilinear coordinates which are such that

$$0 \leq \xi \leq 1, \qquad 0 \leq \eta \leq 1 \tag{2}$$

The surface on which P lies is then a doubly-ruled surface and is the same surface regardless of which corner of the quadrilateral is named A.

If $\boldsymbol{n}_p$ is the unit normal vector at P to the base and $\mathrm{d}S_p$ is an elementary element of area of the base about P corresponding to small changes $\mathrm{d}\xi$ and $\mathrm{d}\eta$ in ξ and η, then by using Eq. (1) we have

$$\begin{aligned} \boldsymbol{n}_p \mathrm{d}S_p &= [\mathbf{AB} + \eta(\mathbf{AC} - \mathbf{AB} - \mathbf{AD})] \\ &\quad \times [\mathbf{AD} + \xi(\mathbf{AC} - \mathbf{AB} - \mathbf{AD})]\,\mathrm{d}\xi\mathrm{d}\eta \\ &= \{[\mathbf{AB} \times \mathbf{AD}] + \xi[\mathbf{AB} \times \mathbf{DC}] + \eta[\mathbf{BC} \times \mathbf{AD}]\}\,\mathrm{d}\xi\mathrm{d}\eta \end{aligned} \tag{3}$$

The volume V_{RABCD} of the pyramid RABCD is therefore

$$\begin{aligned} V_{\mathrm{RABCD}} &= \frac{1}{3}\int_{\mathrm{ABCD}} \mathbf{RP} \cdot \boldsymbol{n}_p \mathrm{d}S_p \\ &= \frac{1}{6}\mathbf{RA} \cdot [\mathbf{DB} \times \mathbf{AC}] + \frac{1}{12}\mathbf{AC} \cdot [\mathbf{AD} \times \mathbf{AB}] \end{aligned} \tag{4}$$

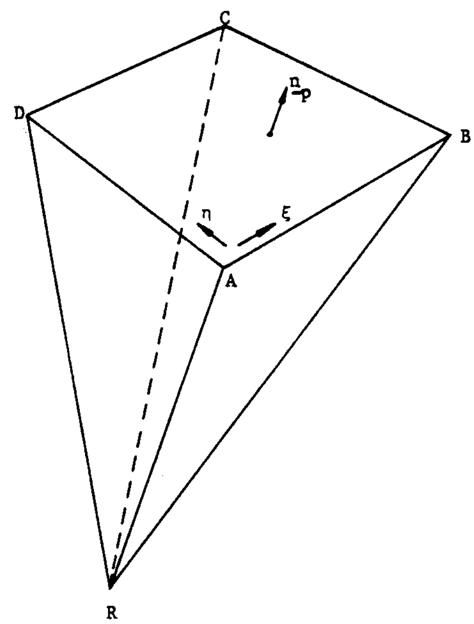

Fig. 1 Pyramid with vertex R and base formed of a doubly-ruled surface bounded by the quadrilateral ABCD.

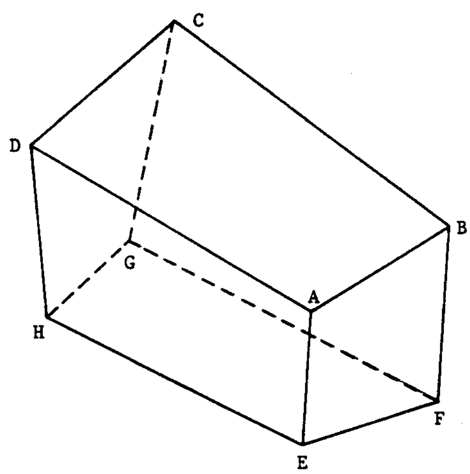

Fig. 2 General hexahedron ABCDEFGH.

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